2015年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2015

学科試験　問題

EXAMINATION QUESTIONS

（学部留学生）

UNDERGRADUATE STUDENTS

数　　学（B）

MATHEMATICS (B)

注意　☆試験時間は６０分。

PLEASE NOTE: THE TEST PERIOD IS **60 MINUTES**.

MATHEMATICS (B) **ANSWER SHEET** (2015)

Nationality		No.		Marks	
Name	(Please print full name, underlining family name)				

<table>
<tr><th colspan="2">Question No.</th><th colspan="6">Your Response</th></tr>
<tr><td rowspan="5">1</td><td>(1)</td><td colspan="2">①</td><td colspan="2">②</td><td colspan="2">③</td></tr>
<tr><td>(2)</td><td colspan="6"></td></tr>
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Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

1. Fill in the blanks with the correct numbers.

(1) If the equation $\log_{10}(ax)\log_{10}(bx) + 1 = 0$ with $a > 0$, $b > 0$ constants has a solution $x > 0$, it follows that $\dfrac{b}{a} \geq$ ① or ② $\geq \dfrac{b}{a} >$ ③.

(2) If $\cos\theta = \sqrt{\dfrac{1}{2} + \dfrac{1}{2\sqrt{2}}}$ and $\sin\theta = -\sqrt{\dfrac{1}{2} - \dfrac{1}{2\sqrt{2}}}$ with $0 \leq \theta < 2\pi$, it follows that $2\theta =$ ☐ π.

(3) If $y = \log_2(x + \sqrt{x^2+1})$, then $2^y - 2^{-y} =$ ☐ x.

(4) The function $f(x) = \log_2(\log_3(\log_2(\log_3(\log_2 x))))$ has the interval $x >$ ☐ as its maximum domain on real numbers.

(5) The total number of subtractions that result in 11111 remaining after a four-digit number has been subtracted from a five-digit number and the digits 1 through 9 have all been used is ☐.

2. Draw a circle C on a plane. Put n distinct points on the circumference of C. Join any two of the n points with a chord. Suppose that no three chords intersect at any one point inside C. Let c_n be the number of such chords, i_n that of intersections of the chords inside C, and r_n that of regions in C bounded by an arc and/or some chords. Fill in the blanks with the answers to the following questions.

(1) Find $c_1, c_2, c_3, c_4, c_5, c_6$ and write their values in this order.

(2) Find $i_1, i_2, i_3, i_4, i_5, i_6$ and write their values in this order.

(3) Find $r_1, r_2, r_3, r_4, r_5, r_6$ and write their values in this order.

(4) Express c_n, i_n, r_n with binomial coefficients in terms of n.
(For binomial coefficients $\binom{m}{k}$, note that $\binom{m}{k} = 0$ if $m < k$.)

(1) $c_1,\ldots,c_6$: ① [ ], ② [ ], ③ [ ], ④ [ ], ⑤ [ ], ⑥ [ ].

(2) $i_1,\ldots,i_6$: ① [ ], ② [ ], ③ [ ], ④ [ ], ⑤ [ ], ⑥ [ ].

(3) $r_1,\ldots,r_6$: ① [ ], ② [ ], ③ [ ], ④ [ ], ⑤ [ ], ⑥ [ ].

(4) $c_n =$ ① [ ], $i_n =$ ② [ ], $r_n =$ ③ [ ].

3. Let C be the plane curve $y = f(x)$ defined by the cubic function $f(x) = x^3 - 4x^2 + ax + b$ with a, b real constants. Fill in the blanks with the answers to the following questions.

(1) When C is tangent to the x-axis at $x = 3$, what are a and b?

(2) When (1) holds, find all x such that C has points in common with the x-axis.

(3) When (1) holds, calculate the area S of the limited region bounded by C and the x-axis.

(1) $a =$ ① ________, $b =$ ② ________.

(2) $x =$ ________.

(3) $S =$ ________.