2012 年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2012

学科試験　　問題

EXAMINATION QUESTIONS

（学部留学生）

UNDERGRADUATE STUDENTS

数　　学（B）

MATHEMATICS(B)

注意　☆試験時間は 60 分。

PLEASE NOTE : THE TEST PERIOD IS 60 MINUTES.

(2012)

MATHEMATICS (B)

Nationality		No.	
Name	(Please print full name,underlining family name)		

Marks	

1. Fill in the blanks with the correct numbers.

(1) If $k = \dfrac{\log_7 9}{\log_7 4}$, then $2^{5k} =$ $\square$.

(2) If $\dfrac{2^x - 2^{-x}}{2^x + 2^{-x}} = \dfrac{1}{3}$, then $x =$ $\square$.

(3) Let $f(x) = -px + 2, g(x) = 5x + 1$. If $f(g(x)) = g(f(x))$, then $p =$ $\square$.

(4) When $\cos 2x + 3\cos x - 1 = 0 \ \ (0 \leq x \leq \pi)$, then $x =$ $\square$.

(5) Let $p = n^2 - 18n + 77$ for a natural number n. If $p > 0$ and p is prime, then $p =$ $\square$.

2. Take a point $P(a, e^{-a})$ $(a > -1)$ on the curve $C : y = e^{-x}$. Let $S(a)$ be the area of the triangle surrounded by the tangent line to C at P, the x-axis and the y-axis.

(1) Find the function $S(a)$.

(2) Find the maximum of $S(a)$.

3. Let a_n be the n-th term of the following sequence

$$\frac{1}{1}, \frac{1}{4}, \frac{3}{4}, \frac{1}{9}, \frac{3}{9}, \frac{5}{9}, \frac{1}{16}, \frac{3}{16}, \frac{5}{16}, \frac{7}{16}, \frac{1}{25}, \cdots.$$

(1) Find a_{50}.

(2) Find the sum $\sum_{n=1}^{50} a_n$.

(3) Find the maximal n satisfying $a_n \geq \frac{1}{10}$.