2015年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2015

学科試験　問題

EXAMINATION QUESTIONS

(学部留学生)

UNDERGRADUATE STUDENTS

数　　学（A）

MATHEMATICS (A)

注意　☆試験時間は６０分。

PLEASE NOTE: THE TEST PERIOD IS **60 MINUTES**.

MATHEMATICS(A) The Answer Sheet (2015)

Nationality		No.	
Name	(Please print full name, underlining family name)		

Marks	

Problem	Question Number	Your Response
1	[1-1]	
	[1-2]	
	[1-3]	
	[1-4]	
	[1-5]	
	[1-6]	
	[1-7]	
	[1-8]	
	[1-9]	
	[1-10]	
	[1-11]	
	[1-12]	
	[1-13]	
	[1-14]	
	[1-15]	
2	[2-1]	
	[2-2]	
	[2-3]	
	[2-4], [2-5], [2-6]	
	[2-7]	
	[2-8]	
	[2-9]	
3	[3-1]	
	[3-2]	
	[3-3]	

MATHEMATICS(A) (2015)

Nationality		No.	
Name	(Please print full name, underlining family name)		

Marks	

1. Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

(1) $2x^2 - 5x$ for the values $1 \leq x \leq 4$ takes its maximum [1-1] at $x =$ [1-2], and its minimum [1-3] at $x =$ [1-4].

(2) Two persons A, B simultaneously toss their individual coins, and win 1 point if the head is face-up, and 0 point if the tail is face-up. The probability that the points of A exceed the points of B after three tosses is [1-5].

(3) When $a = \sqrt{5} + \sqrt{3}$ and $b = \sqrt{5} - \sqrt{3}$, $\dfrac{a}{b} + \dfrac{b}{a}$ is equal to an integer [1-6].

(4) The negation of proposition "$x \neq 0$ and $y \neq 0$" is "x [1-7] 0 [1-8] y [1-9] 0".

(5) There exist two circles that go through two points $(1, 3), (2, 4)$ and are tangent to the y-axis. Letting the radii of the circles be a, b implies that $ab =$ [1-10].

(6) For the equation $|2x - 1| + |x - 2| = 2$, the minimum of x is $x =$ [1-11] and the maximum is $x =$ [1-12].

(7) For $\omega = \dfrac{1 + \sqrt{3}\,i}{2}$, it holds that $\omega^5 =$ [1-13] + [1-14] i, where i denotes the imaginary unit. Note that the answers are real numbers.

(8) For the sequence $\{a_n\}$ defined by $a_{n+1} - a_n = 2n$, $a_1 = 0$ where n is a positive integer, the general term is $a_n =$ [1-15].

2. A circle O is circumscribed around a triangle ABC, and its radius is r. The angles of the triangle are $\angle CAB = a$, $\angle ABC = b$, and $\angle BCA = c$.

(1) The lengths of arcs AB, BC, and CA are expressed by using a, b, c, and r as [2-1], [2-2], and [2-3], respectively.

(2) The area of $\triangle$ ABC is expressed by using a, b, c, and r as

$$\frac{r^2}{2}\left\{\sin\left(\boxed{[2\text{-}4]}\right)+\sin\left(\boxed{[2\text{-}5]}\right)+\sin\left(\boxed{[2\text{-}6]}\right)\right\}.$$

(3) When $a = 75°, b = 60°, c = 45°$, and $r = 1$, the lengths of sides AB, BC, and CA are calculated as [2-7], [2-8], and [2-9] without using trigonometric functions.

3. If a function $f(x)$ satisfies the following equation

$$\int_a^x f(t)\,dt = 3x^2 + (a+8)x + 4,$$

then the constant a is [3-1] and the function $f(x)$ is $f(x) =$ [3-2]. In this obtained function, the minimum of the integral $\int_a^x f(t)\,dt$ is [3-3].