2014年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2014

学科試験　問題

EXAMINATION QUESTIONS

（学部留学生）

UNDERGRADUATE STUDENTS

数　　学（A）

MATHEMATICS (A)

注意　☆試験時間は６０分。

PLEASE NOTE: THE TEST PERIOD IS **60 MINUTES**.

MATHEMATICS(A) The Answer Sheet (2014)

Nationality		No.	
Name	(Please print full name, underlining family name)		

Marks	

Problem	Question Number	Your Response
1	[1-1]	
	[1-2]	
	[1-3]	
	[1-4]	
	[1-5]	
	[1-6]	
	[1-7]	
	[1-8]	
2	[2-1]	
	[2-2]	
3	[3-1]	
	[3-2]	
	[3-3]	
	[3-4]	
	[3-5]	
	[3-6]	
	[3-7]	
	[3-8]	

Nationality		No.	
Name	(Please print full name, underlining family name)		

Marks	

1. Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

(1) Let a and b be an integer part and an decimal fraction of $\sqrt{7}$, respectively. Then the integer part of $\frac{a}{b}$ is [1-1] .

(2) Consider a cone with a diameter of 12 and a height of 8. The volume of an inscribed sphere in the cone is [1-2] .

(3) 5^{29} is an integer with [1-3] places by assuming that $\log_{10} 2 = 0.3010$.

(4) There is a circle with a radius of 2 where the center is at the origin and a line $3x + 4y - 12 = 0$ in the plane. The minimum distance between a point on the circle and a point on the line is [1-4] .

(5) If the series $\{a_k\}$ satisfies that $a_1 = 1, a_2 = 2$, and $a_k - 4a_{k-1} + 3a_{k-2} = 0$ $(k \geq 3)$, then $a_k = \dfrac{1 + \boxed{[1\text{-}5]}}{\boxed{[1\text{-}6]}}$ $(k \geq 1)$.

(6) Let $f(x) = ax + b$ be a linear function. If the equation

$$\int_{-m/2}^{m} f(x)dx = \frac{m(m+1)}{2}$$

holds for any positive m, then $f(x) = \dfrac{\boxed{[1\text{-}7]}\,x + \boxed{[1\text{-}8]}}{3}$.

2. Consider a semicircle with a diameter AB where the length is 4, and a point C on the circular arc. Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

(1) The maximum of the area of the triangle ABC is [2-1] .

(2) If the area of the triangle ABC is a half of the maximum and point C is nearer to point A than point B, then the angle $\angle CAB$ is [2-2] .

3. Consider a function

$$y = \left(x^3 + \frac{1}{x^3}\right) - 6\left(x^2 + \frac{1}{x^2}\right) + 3\left(x + \frac{1}{x}\right)$$

defined in $x > 0$.

(1) Letting $t = x + \dfrac{1}{x}$ gives

$$y = \boxed{\text{[3-1]}}\, t^3 + \boxed{\text{[3-2]}}\, t^2 + \boxed{\text{[3-3]}}\, t + \boxed{\text{[3-4]}}\,.$$

Here it holds that

$$t = x + \frac{1}{x} \geq \boxed{\text{[3-5]}}\,.$$

(2) When $t = \boxed{\text{[3-6]}}$, that is, $x = \boxed{\text{[3-7]}}$, y has the minimum value $\boxed{\text{[3-8]}}$.