2013 年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2013

学科試験　問題
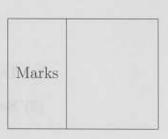EXAMINATION QUESTIONS

（学部留学生）
UNDERGRADUATE STUDENTS

数　　学（A）
MATHEMATICS（A）

注意　☆試験時間は６０分。
PLEASE NOTE: THE TEST PERIOD IS **60 MINUTES.**

MATHEMATICS(A) (2013)

Nationality		No.	
Name	(Please print full name, underlining family name)		

Marks

1. Fill in the blanks with the correct answers.

(1) The radius of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ is ☐ .

(2) If three straight lines $x + 2y - 1 = 0$, $x - y + 2 = 0$ and $ax - y + 3 = 0$

meet at one point, then $a =$ ☐ .

(3) The solution of the inequality $\sqrt{5 - x} < x + 1$ is

(a) ☐ $< x \leqq$ (b) ☐ .

(4) Let α and β be two solutions of the equation $x^2 - x + 4 = 0$.

Then $\dfrac{\beta}{\alpha} + \dfrac{\alpha}{\beta} =$ ☐ .

(5) For $a = \dfrac{\sqrt{10} + \sqrt{2}}{2}$ and $b = \dfrac{\sqrt{10} - \sqrt{2}}{2}$, we have

$\log_2(a^2 + ab + b^2) =$ ☐ .

2. Suppose that $f(x) = ax + b$ satisfies the three conditions:

$$\int_0^2 f(x)dx = 2, \quad \int_0^2 \{f(x)\}^2 dx = 4 \quad \text{and} \quad f(0) > 0.$$

(1) Determine $f(x)$.

(2) Set $g(x) = f(x) + c$. When c varies, find the minimumn of the integral

$$\int_0^2 \{g(x)\}^2 dx.$$

Fill in the blanks with the correct answers.

(1) $f(x) =$ ☐

(2) ☐

3. Take a line segment AB with a length 6. Consider a semicircle with AB as the diameter. Let P be a point on the arc $\overset{\frown}{AB}$. Let $x = \angle ABP$.

   (1) Express the area of the $\triangle APB$ in terms of x.

   (2) Find the range of x for which the area of the $\triangle APB \geq \dfrac{9\sqrt{2}}{2}$.

   (3) If the point P is so chosen that $PA+PB=3\sqrt{6}$ holds, find the area of the $\triangle APB$.

Fill in the blanks with the correct answers.

(1)

(2)

(3)