2010年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE

GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2010

学科試験　　問題

EXAMINATION QUESTIONS

(学部留学生)

UNDERGRADUATE STUDENTS

数　　学（A）

MATHEMATICS(A)

注意　☆試験時間は60分。

PLEASE NOTE : THE TEST PERIOD IS 60 MINUTES.

Q 1 The quadratic function which takes the value 41 at $x = -2$ and the value 20 at $x = 5$ and is minimized at $x = 2$ is

$$y = \boxed{\textsf{A}}\,x^2 - \boxed{\textsf{B}}\,x + \boxed{\textsf{C}}.$$

The minimum value of this function is $\boxed{\textsf{D}}$.

Q 2 Consider the integral expression in x

$$P = x^3 + x^2 + ax + 1,$$

where a is a rational number.

At $a =$ **A** the value of P is a rational number for any x which satisfies the equation $x^2 + 2x - 2 = 0$, and in this case the value of P is **B**.

Q 3 For each of **A** ~ **D** in the following statements, choose the most appropriate expression from among ⓪~⑨ below.

Consider two conditions $x^2 - 3x - 10 < 0$ and $|x-2| < a$ on a real number x, where a is a positive real number.

(i) A necessary and sufficient condition for $x^2 - 3x - 10 < 0$ is that **A** $< x <$ **B** .

(ii) The range of values of a such that $|x-2| < a$ is a necessary condition for $x^2 - 3x - 10 < 0$ is **C** .

(iii) The range of values of a such that $|x-2| < a$ is a sufficient condition for $x^2 - 3x - 10 < 0$ is **D** .

⓪ 2	① 5	② -2	③ -5
④ $a \geqq 2$	⑤ $a \geqq 3$	⑥ $a \geqq 4$	
⑦ $0 < a \leqq 2$	⑧ $0 < a \leqq 3$	⑨ $0 < a \leqq 5$	

Q 4 We are to find the probability that when three dice are rolled at the same time, the largest value of the three numbers rolled is 4.

Let A be the outcome in which the largest number is 4, let B be the outcome in which the largest number is 4 or less, and let C be the outcome in which the largest number is 3 or less.

Let $P(X)$ denote the probability that the outcome of an event is X. Then

(1) $P(B) = \boxed{\text{A}}$, $P(C) = \boxed{\text{B}}$.

(2) Since $B = A \cup C$ and the outcomes A and C are mutually exclusive, it follows that

$$P(A) = \boxed{\text{C}}.$$

Q 5 We are to find the value of $x^4 + y^4 + z^4$ when x, y and z are real numbers which satisfy the following three equalities:

$$\begin{cases} x + y + z = 3 \\ x^2 + y^2 + z^2 = 9 \\ xyz = -2 \end{cases}$$

Firstly, it follows from the first two equalities that

$$xy + yz + zx = \boxed{\mathsf{A}}.$$

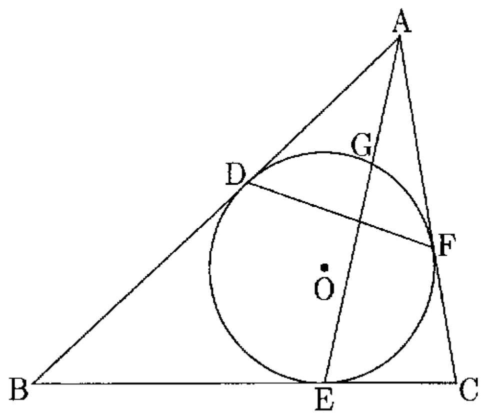
Next, using

$$(x^2 + y^2 + z^2)^2 = x^4 + y^4 + z^4 + \boxed{\mathsf{B}}\left\{(xy)^2 + (yz)^2 + (zx)^2\right\},$$

we have

$$x^4 + y^4 + z^4 = \boxed{\mathsf{C}}.$$

Q 6 Consider a triangle ABC, where $\angle A = 60°$. Let O be the inscribed circle of triangle ABC, as shown in the figure. Let D, E and F be the points at which circle O is tangent to the sides AB, BC and CA. And let G be the point of intersection of the line segment AE and the circle O. Set $x = AD$.



(1) Let $\triangle ADF$ be the area of the triangle ADF. Then

$$\frac{\triangle ADF}{AG \cdot AE} = \boxed{\text{A}}.$$

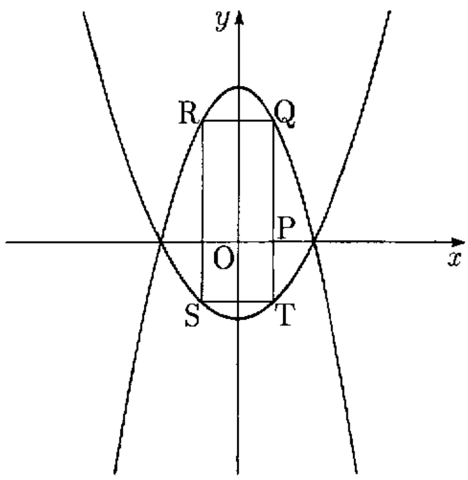
(2) When $BD = 4$ and $CF = 2$, then $BC = \boxed{\text{B}}$ and x satisfies the equation

$$x^2 + \boxed{\text{C}}\,x - \boxed{\text{D}} = 0.$$

Solving this equation, we have

$$AD = \boxed{\text{E}}.$$

Q 7 Consider a rectangle QRST such that the side QR is parallel to the x-axis, the two vertices Q and R are on the $y > 0$ portion of the graph of the quadratic function $y = -x^2 + 4$, and the two other vertices S and T are on the $y < 0$ portion of the graph of the quadratic function $y = \frac{1}{2}x^2 - 2$ as shown in the figure to the right. Let P be the point of intersection of the side QT and the x-axis. Let ℓ be the length of the perimeter of this rectangle. We are to find the x-coordinate of the point P where ℓ is maximized and also to find the maximum value of ℓ.



Let P be $(\alpha, 0)$, where $0 < \alpha < 2$. Since

$$\mathrm{PQ} = \boxed{\mathsf{A}} - \alpha^2, \quad \mathrm{QR} = \boxed{\mathsf{B}}\,\alpha, \quad \mathrm{PT} = \boxed{\mathsf{C}} - \frac{1}{\boxed{\mathsf{D}}}\alpha^2,$$

we have

$$\ell = \boxed{\mathsf{E}} + \boxed{\mathsf{F}}\,\alpha - \boxed{\mathsf{G}}\,\alpha^2.$$

Therefore, when $\alpha = \boxed{\mathsf{H}}$, ℓ is maximized and its maximum value is $\boxed{\mathsf{I}}$.