## 2009年度日本政府(文部科学省)奨学金留学生選考試験

## QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2009

学科試験 問題

**EXAMINATION QUESTIONS** 

(学部留学生)

UNDERGRADUATE STUDENTS

数 学(A)

MATHEMATICS (A)

注意 ☆試験時間は60分。

PLEASE NOTE: THE TEST PERIOD IS 60 MINUTES.

MATHEMATICS (A)

Nationality

No.

(Please print full name, underlining family name)

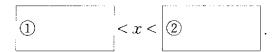
Name

Marks

- 1 Fill in the blanks with the correct numbers.
- (1) Let  $\alpha$  and  $\beta$  be solutions of  $3x^2 x 3 = 0$ .

Then 
$$\alpha^2 + \beta^2 =$$
 .

(2) The solution of the inequality  $-x < x^2 < 2x + 1$  is



(3) Let  $\sin \alpha = \frac{1}{\sqrt{5}}$  (0 <  $\alpha$  < 90°) and  $\cos \beta = \frac{3}{\sqrt{10}}$  (0 <  $\beta$  < 90°).

Then  $\sin(\alpha + \beta) = \boxed{}$ .

(4) Let n be a natural number. If  $3^n < 2^{100} < 3^{n+1}$ , then  $n = 2^{n+1}$ .

Use  $\log_3 2 = 0.631$ .

(5) The total number of pairs of integers (x, y) which satisfy the equation

$$x^2 - 4xy + 5y^2 + 2y - 4 = 0$$
 is

2 Let 
$$f(a) = \int_0^2 |x(x-a)| dx$$
 for  $0 \le a \le 2$ .

- (1) Find the function f(a).
- (2) Find the minimum of f(a).

3 Let a be a real number such that 1 < a < 2.  $\{a_n\}$  is the sequence defined by

$$a_1 = a$$
,  $a_{n+1} = |a_n| - 1$   $(n = 1, 2, 3 \cdots)$ .

And put  $S_n = a_1 + a_2 + \cdots + a_n$ .

- (1) Find  $a_4$ ,  $a_5$ ,  $a_6$ ,  $a_7$ .
- (2) Find  $S_2$ ,  $S_4$ ,  $S_6$ .
- (3) When n = 2m, where m is an integer  $\ge 1$ , express  $S_n$  in terms of a and m.
- (4) When n = 2m + 1, where m is an integer  $\ge 1$ , express  $S_n$  in terms of a and m.